

**Adaptive Backstepping Sliding Mode Control for Roll Channel of Launch Vehicle**

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Research Article

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**Abstract**

This paper presents a comparative study of Backstepping Control, Adaptive Backstepping Control and Adaptive Backstepping Sliding Mode Control on Roll channel of Launch vehicle. Backstepping Control design involves a systematic construction of both feedback control laws and associated Lyapunov functions. The Backstepping design methodology fails in the presence of parameter uncertainties. With Adaptive Backstepping, stabilization is achieved in the presence of unknown parameters. Robustness of Adaptive Backstepping Control can be increased by designing a sliding surface to the associated Lyapunov function. Thus Adaptive Backstepping Sliding Mode Control combines the advantages of both Adaptive Backstepping and Sliding Mode. The comparison of the proposed control schemes is verified by the MATLAB simulation. From the simulation results it is clear that Adaptive Backstepping Sliding Mode Control Scheme gives better and satisfactory responses.

**Keywords:** Terms-Backstepping Control (BS); Adaptive Backstepping Control (ABS); Adaptive Backstepping Sliding Mode Control (ABSSM); Launch Vehicle.

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**Received:** July 31, 2015

**Accepted:** September 03, 2015

**Published:** September 08, 2015

**Citation:** Arun Kumar V V, Laila Beebi M (2015) Adaptive Backstepping Sliding Mode Control for Roll Channel of Launch Vehicle. *Int J Aeronautics Aerospace Res.* 2(5), 58-64. doi: <http://dx.doi.org/10.19070/2470-4415-150007>

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**Introduction**

LAUNCH vehicles are vehicles which are primarily used to carry payloads from the surface of earth to outer space. Launch vehicles are generally classified as: Expendable launch vehicle or Reusable launch vehicle, Orbital launch vehicle or Suborbital launch vehicle, Manned launch vehicle or Unmanned launch vehicle. Expendable launch vehicles are designed for one-time use. They usually separate from their payload and disintegrate during atmospheric re-entry. Reusable launch vehicles are those which can be used more than once, they are designed to perform many missions within their life cycle.

Launch vehicles are highly complex nonlinear systems. Control of launch vehicle using linear control methods reduces the stability of the system since some useful nonlinearities are neglected.

Several control techniques, including Proportional-Integral-Derivative (PID) control law with gain scheduling [1], Trajectory Linearization Control (TLC) [2], Adaptive Neural Network (NN) [3], have been developed under Advanced Guidance and Control (AG&C) project and is used for launch vehicle flight control. However, by means of these control techniques the robustness issues were not satisfactory.

Backstepping offers flexibilities which are not present in other nonlinear designs; one of them is that it avoids the cancellation of useful nonlinearities. In [4] backstepping is applied to flight path angle control. Backstepping is recursive method based on Lyapunov theory. In backstepping the parameter uncertainties are assumed to be constant, thus fails in the presence of uncertainties. Adaptive Backstepping [5-8] makes use of dynamic parameter update laws to deal with parametric uncertainties. Moreover the stability of a system may be affected by some bounded disturbances and high rate of adaptation which can be overcome by incorporating a robust control. Sliding Mode control method has strong robustness, especially when the system contains uncertainties. In Adaptive Backstepping Sliding Mode Control design [9], it involves a systematic construction of both feedback control laws and associated Lyapunov functions and the robustness of the controller is increased by incorporating a sliding surface in the parameter updation law.

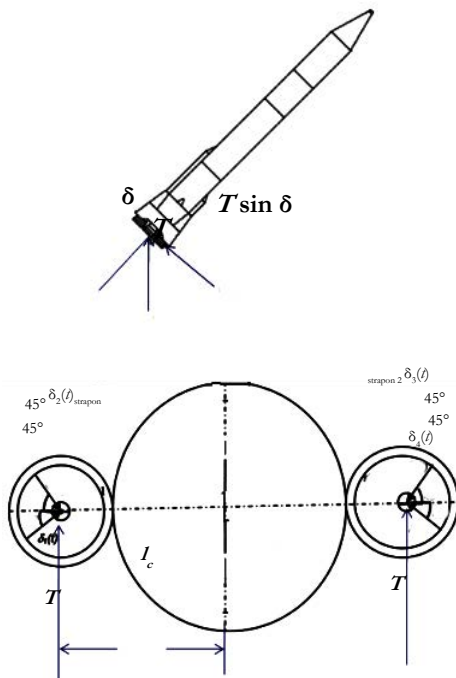
In this paper Backstepping, Adaptive Backstepping and Adaptive Backstepping Sliding Mode Controls have been implemented on the roll channel to control the roll angle of the launch vehicle. The Adaptive Backstepping control method possesses very good adaptability and Sliding Mode control method has strong robustness, thus Adaptive Backstepping Sliding Mode Control combines the merits of both the controllers. Comparing with Backstepping and Adaptive Backstepping Control methods, the Adaptive Backstepping Sliding Mode Control method has more

strong robustness and good adaptability, the effectiveness of the above proposed control schemes is verified by the MATLAB simulation.

The paper has been organized as follows: Section II deals with the modeling of roll dynamics. Section III deals with Backstepping Controller design. Section IV deals with the design of Adaptive Backstepping Control. Section V deals with the design of Adaptive Backstepping Sliding Mode Control. In Section VI the Simulation results are shown with some discussions on it. Section VII is the Conclusion.

### Modelling Roll Dynamics

Figure 1. Launch vehicle roll control.



As shown in the figure there are two thrusters. By varying the thruster angle launch vehicle direction in roll channel is controlled. The tilting force to cause roll is in the direction shown as  $T \sin \delta$ . The total torque is due to thrusts of both the thrusters. Then the torque equation can be written as:

$$I \ddot{\theta} = 2Tl_c \sin \delta$$

$$\ddot{\theta} = \frac{2Tl_c}{I} \sin \delta = \mu_c \sin \delta \quad \text{---- (1)}$$

where  $T$  = force due to each thruster,  $l_c$  = length between the main motor and each thruster,  $\delta$  = thruster angle,  $I$  = moment of inertia of the vehicle,  $\theta$  = roll angle.

### Backstepping Control Design

Backstepping is a recursive procedure which breaks a design problem for the full system into sequence of design problems for lower order systems. Backstepping designs by breaking down complex nonlinear systems into smaller subsystems. Then designing control Lyapunov functions and virtual controls for these systems and finally integrating these individual controllers

into an actual controller, by stepping back through the subsystem and reassembling it from its component subsystems.

In this paper Backstepping Control is designed for the roll channel control of the system (1). In order to adopt the backstepping control design, the model (1) can be transformed into the following:

$$x_1 = \theta ; x_2 = \dot{x}_1 = \dot{\theta}; \delta = u \quad \text{---- (2)}$$

$$\dot{x}_1 = x_2 ; \dot{x}_2 = \mu_c \sin \delta \quad \text{---- (3)}$$

#### A. Step 1

The first subsystem is  $\dot{x}_1 = x_2$ . Let the Lyapunov function be

$$V_1(x) = (1/2) x_1^2 \quad \text{---- (4)}$$

$$\dot{V}_1 = x_1 \dot{x}_1 = x_1 x_2 \quad \text{---- (5)}$$

$$x_{2des} = -c_1 x_1, \quad c_1 > 0 \quad \text{---- (6)}$$

$$\dot{V}_1 = -c_1 x_1^2 \quad \text{---- (7)}$$

Where the scalar  $c_1$  is always greater than zero so that  $\dot{V}_1$  is always negative definite.

#### B. Step 2

Error variable,  $z = x_2 - x_{2des} = x_2 + c_1 x_1$  ---- (8)

$$\dot{z} = \dot{x}_2 + c_1 \dot{x}_1 = \mu_c \sin u + c_1 (z - c_1 x_1) \quad \text{---- (9)}$$

Augmented Lyapunov function,

$$V_2(x, z) = (1/2) x_1^2 + (1/2) z^2 \quad \text{---- (10)}$$

$$\dot{V}_2 = x_1 \dot{x}_1 + z \dot{z} = -c_1 x_1^2 + z [\mu_c \sin u + c_1 (z - c_1 x_1)] \quad \text{---- (11)}$$

The desired value of  $u, u_{des}$  is given by

$$u_{des} = \sin^{-1} \frac{1}{\mu_c} [-c_2 z - c_1 (z - c_1 x_1)] \quad \text{---- (12)}$$

### Adaptive Backstepping Control Design

Adaptive backstepping control is a recursive Lyapunov based nonlinear design method, which makes use of dynamic parameter update laws to deal with parametric uncertainties. For applying adaptive backstepping control design on roll channel of system (1) the equations are modified as:

$$\dot{x}_1 = x_2;$$

$$\dot{x}_2 = \mu_c \sin \delta = \phi \sin \delta \quad \text{---- (13)}$$

where  $\Phi = \mu_c$  is the unknown parameter.

**A. Step 1**

The first error variable is defined as

$$e = x_1 - \theta_{sp} \quad \text{--- (14)}$$

where  $\theta_{sp}$  is the desired set point.

Using the Lyapunov function

$$V_1 = 1/2 e^2 \quad \text{--- (15)}$$

$$\dot{V}_1 = e\dot{e} = e(x_2 - \dot{\theta}_{sp}) \quad \text{--- (16)}$$

Using the derivative of the Lyapunov function, the virtual control law can be formulated as:

$$x_{2des} = -k_1 e + \dot{\theta}_{sp} \quad \text{--- (17)}$$

where  $k_1 > 0$  and is a design parameter which guarantees

$$\dot{V}_1 < 0$$

**B. Step 2**

The second error variable  $\xi$  is defined as:

$$\xi = x_2 - x_{2des} \quad \text{--- (18)}$$

By augmenting the Lyapunov function  $V_1$  with the error variable  $\xi$  and the unknown parameters in the system, we get

$$V = \frac{1}{2}e^2 + \frac{1}{2}\xi^2 + \frac{1}{2}\varnothing^2 \quad \text{--- (19)}$$

where  $\varnothing$  is the parameter estimation error of  $\Phi$ , and  $\gamma$  is the adaptation gain.

$$\begin{aligned} \dot{V}_2 &= e\dot{e} + \xi\dot{\xi} + \frac{1}{\gamma}\varnothing\dot{\varnothing} \\ &= -k_1 e^2 + \xi(x_2 - \dot{x}_{2des}) + \frac{1}{\gamma}\varnothing(\dot{\Phi}\Phi) \\ &= -k_1 e^2 + \xi(\hat{\phi}\sin u - \ddot{\theta}_{sp} + k_1 \dot{x}_1 - k_1 \dot{\theta}_{sp}) \\ &\quad + \varnothing(\xi\sin u - \frac{1}{\gamma}\dot{\varnothing}) \end{aligned} \quad \text{--- (20)}$$

The parameter adaptation law is obtained by setting:

$$\xi\sin u - \frac{1}{\gamma}\dot{\varnothing} = 0 \quad \text{--- (21)}$$

Thus the adaptation law is given by:

$$\dot{\phi} = \gamma\xi\sin u \quad \text{--- (22)}$$

The control law is obtained by setting:

$$\hat{\phi}\sin u - \ddot{\theta}_{sp} + k_1 \dot{x}_1 - k_1 \dot{\theta}_{sp} = -k_2\xi \quad \text{--- (23)}$$

where  $k_2$  is the design parameter, such that  $k_2 > 0$ .

Thus control law is given by:

$$u = \sin^{-1} \frac{1}{\hat{\phi}} [-k_2\xi + \ddot{\theta}_{sp} - k_1 x_2 + k_1 \dot{\theta}_{sp}] \quad \text{--- (24)}$$

**Adaptive Backstepping Sliding Mode Control Design**

The Adaptive Backstepping control method possesses very good adaptability but relatively large estimation time and over parameterization are the two disadvantages of Adaptive Backstepping Control. Sliding Mode Control method has strong robustness, which restrains the uncertainty through the design of the sliding surface. Thus Adaptive Backstepping Sliding Mode Control [9] combines the advantages of both Adaptive Backstepping and Sliding Mode Control schemes.

For applying Adaptive Backstepping Sliding Mode Control design on roll channel of system (1) the equations are modified as:

$$\dot{x}_1 = x_2; x_2 = \mu_c \sin \delta = \phi \sin \delta \quad \text{--- (25)}$$

where  $\Phi = \mu_c$  is the unknown parameter.

**A. Step 1**

The first error variable is defined as

$$e = x_1 - \theta_{sp} \quad \text{--- (26)}$$

where  $\theta_{sp}$  is the desired set point.

Using the Lyapunov function

$$V_1 = (1/2) e^2 \quad \text{--- (27)}$$

$$\dot{V}_1 = e\dot{e} = e(x_2 - \dot{\theta}_{sp}) \quad \text{--- (28)}$$

Using the derivative of the Lyapunov function, the virtual control law can be formulated as:

$$x_{2des} = -c_1 e + \dot{\theta}_{sp} \quad \text{--- (29)}$$

where  $c_1 > 0$  and is a design parameter which guarantees  $\dot{V}_1 < 0$ .

**B. Step 2**

The second error variable  $\xi$  is defined as:

$$\xi = x_2 - x_{2des} \quad (30)$$

By augmenting the Lyapunov function  $V_1$  with the error variable  $\xi$  and the unknown parameters in the system, we get:

$$V_2 = 1/2 e^2 + 1/2 \xi^2 + 1/2 \gamma \hat{\phi}^2 + 1/2 \sigma^2 \quad (31)$$

where  $\hat{\phi}$  is the parameter estimation error of  $\Phi$ ,  $\sigma$  is the sliding surface which is expressed by  $\sigma = k_f e + \xi$ , such that the scalar  $k_f > 0$  and  $\gamma > 0$  is any real constant called parameter adaptation gain.

$$\begin{aligned} \dot{V}_2 &= e \dot{e} + \xi \dot{\xi} + \frac{1}{\gamma} \hat{\phi} \dot{\phi} + \sigma \dot{\sigma} \\ &= -c_1 e^2 + \xi (\dot{x}_2 - \dot{x}_{2des}) + \frac{1}{\gamma} \hat{\phi} (\dot{\phi} - \dot{\hat{\phi}}) + \sigma (k_f \dot{e} + \dot{\xi}) \\ &= -c_1 e^2 + \xi [\hat{\phi} \sin u - \ddot{\theta}_{sp} + c_1 \dot{x}_1 - c_1 \dot{\theta}_{sp}] + \\ &\quad \sigma [\hat{\phi} \sin u - \ddot{\theta}_{sp} + (k_1 + c_1) \dot{x}_1 - (k_1 + c_1) \dot{\theta}_{sp}] \\ &\quad + \hat{\phi} (\xi \sin u - \frac{1}{\gamma} \dot{\hat{\phi}} + \sigma \sin u) \end{aligned} \quad (32)$$

The parameter adaptation law is obtained by setting:

$$\xi \sin u - \frac{1}{\gamma} \dot{\hat{\phi}} + \sigma \sin u = 0 \quad (33)$$

Thus the adaptation law is given by:

$$\dot{\hat{\phi}} = \gamma (\xi + \sigma) \sin u \quad (34)$$

The control law is obtained by setting:

$$\hat{\phi} \sin u - \ddot{\theta}_{sp} + c_1 \dot{x}_1 - c_1 \dot{\theta}_{sp} = -c_2 \xi$$

$$\hat{\phi} \sin u - \ddot{\theta}_{sp} + (k_1 + c_1) \dot{x}_1 - (k_1 + c_1) \dot{\theta}_{sp} = -c_3 \sigma \quad (35)$$

where  $c_2$  and  $c_3$  the design parameters, such that  $c_2 > 0$  and  $c_3 > 0$ .

Thus control law is given by:

$$u = \sin^{-1} \frac{1}{\hat{\phi}} [2\ddot{\theta}_{sp} - c_2 \xi - c_3 \sigma - (k_1 + 2c_1) \dot{x}_2 + (k_1 + 2c_1) \dot{\theta}_{sp}] \quad (36)$$

With equations (34) and (35), the derivatives of the augmented Lyapunov function becomes,

$$\dot{V}_2 = -c_1 e^2 - c_2 \xi^2 - c_3 \sigma^2 \quad (37)$$

making the system globally asymptotically stable.

### Simulation Results and Discussions

Figure.2 shows the regulation of the roll angle using BS for different values of  $C_1$  and  $C_2$ , with an initial condition of 20 degree. Figure.3 shows the variation of roll rate with time for the same initial condition. Figure.4 shows the tracking of roll angle. Figure.5 shows the regulation of roll angle using ABS with 20 degree initial condition and for different values of  $C_1$  and  $C_2$ . As the value of  $C_1$  and  $C_2$  is increased the settling time decreases but for each case there is a small overshoot. Figure.6 shows the variation of roll angle for different values of  $C_1$  and  $C_2$ . Figure.7 shows the tracking of roll angle using ABS. Figure.8 shows the regulation of roll angle using ABSSM with 20 degree initial condition and for different values of  $C_1$ ,  $C_2$  and  $K_1$ . As the value of constants is increased the settling time decreases without any overshoot. Figure.9 shows the regulation of roll rate

Figure 2. Regulation of Roll angle using Backstepping Control.

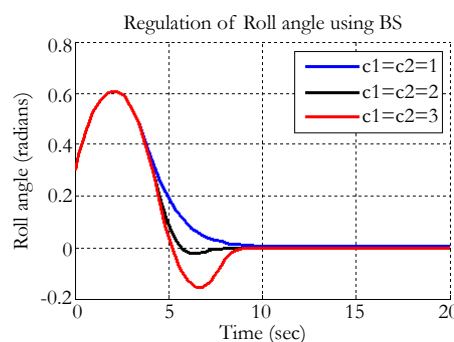


Figure 3. Regulation of Roll rate using Backstepping Control.

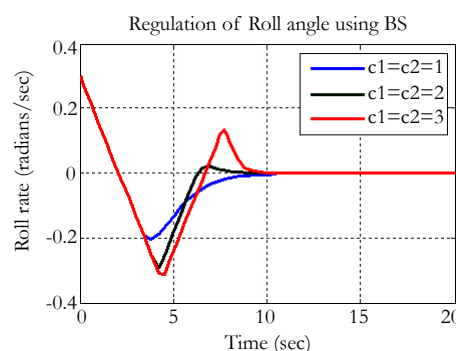


Figure 4. Tracking of Roll angle using Backstepping Control.

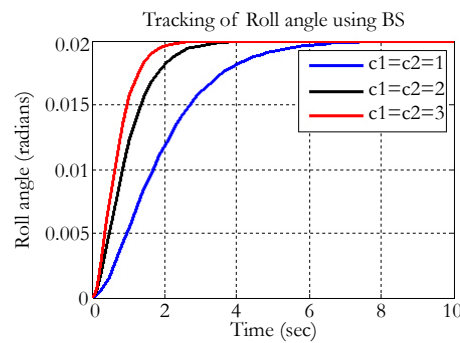


Figure 5. Regulation of Roll angle using Adaptive Backstepping Control.

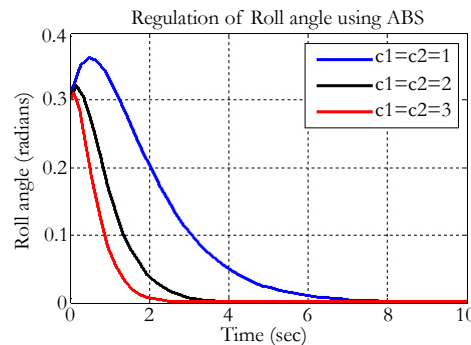


Figure 6. Regulation of Roll rate using Adaptive Backstepping Control.

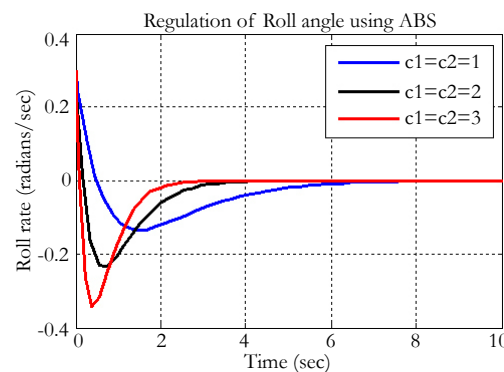
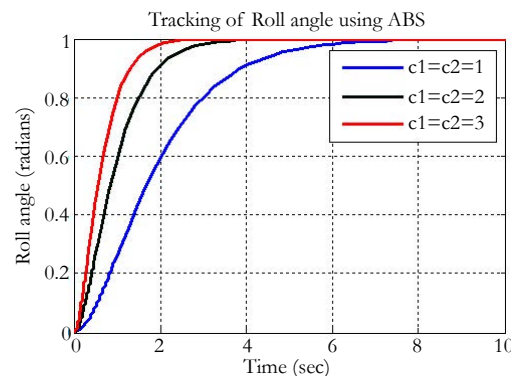


Figure 7. Tracking of Roll angle using Adaptive Backstepping Control.



using ABSSM. Figure.10 shows the tracking of roll angle using ABSSM. Figure.11 shows the comparison of regulation of roll angle using all the three controllers with  $C_1=C_2=1$ . Figure.12 shows the comparison of regulation of roll rate using all the three controllers with  $C_1=C_2=1$ . Figure 13 shows the comparison of tracking of roll angle using all the three controllers with  $C_1=C_2=1$ . Figure 14 shows the disturbance analysis of the three controllers when they are subjected to a disturbance of 10 degree. From

the simulation results, it is clear that the Adaptive Backstepping Sliding Mode Controller gives a better response than others.

### Conclusion

In this paper Backstepping, Adaptive Backstepping and Adaptive Backstepping Sliding Mode Control have been designed for the roll channel of launch vehicle. While designing Backstepping non

Figure 8. Regulation of Roll angle using Adaptive Backstepping Sliding Mode Control.

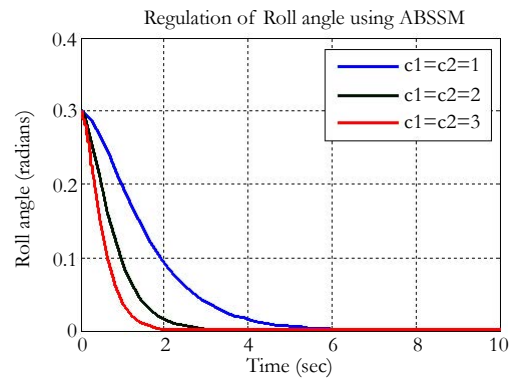


Figure 9. Regulation of Roll rate using Adaptive Backstepping Sliding Mode Control.

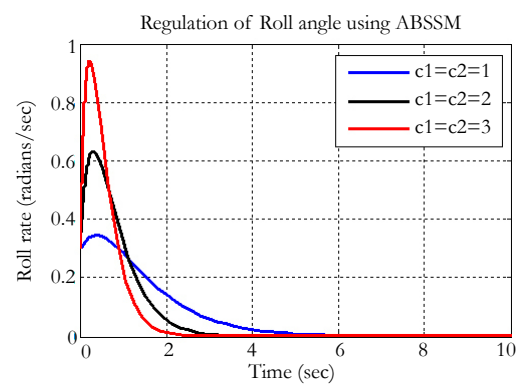


Figure 10. Tracking of Roll angle using Adaptive Backstepping Sliding Mode Control.

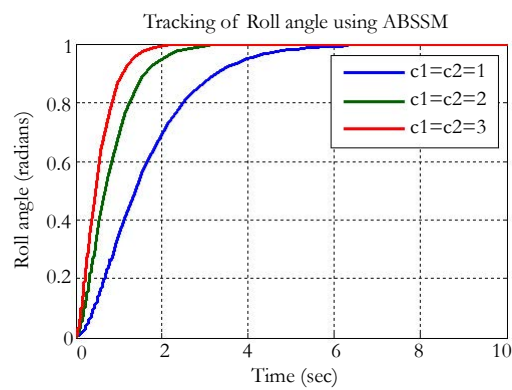
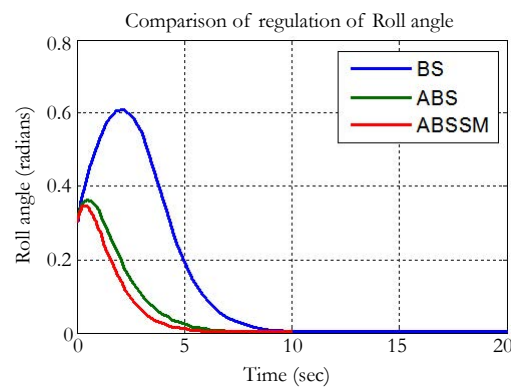
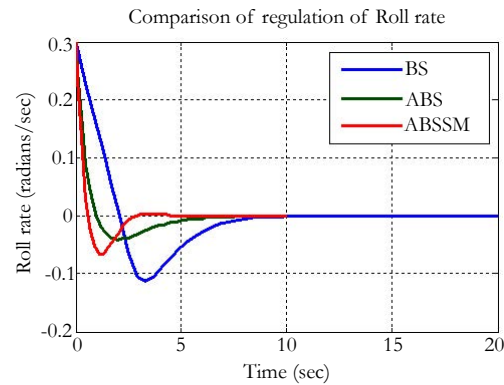


Figure 11. Comparison of Regulation of Roll angle.

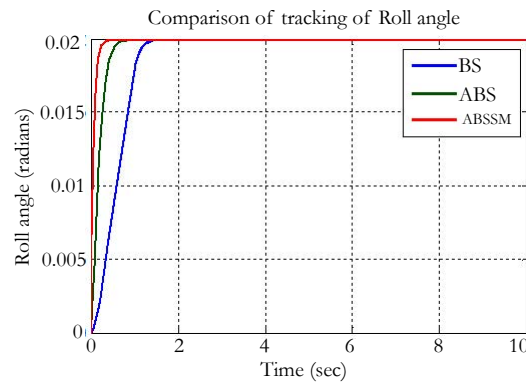




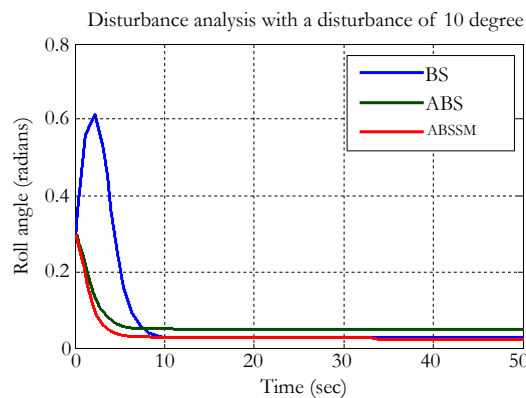
**Figure 12. Comparison of Regulation of Roll rate.**



**Figure 13. Comparison of Tracking of Roll angle.**



**Figure 14. Disturbance analysis.**



linearities affecting the system were taken as constant where as in Adaptive Backstepping design uncertainties associated with the system is considered. Simulation results shows that Adaptive Backstepping Sliding Mode design gives comparatively better and satisfactory responses of the three controllers. Thus Adaptive Backstepping Sliding Mode Controllers displays strong robustness and adaptability.

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