

Flutter Analysis of A Laminated Composite Plate with Temperature Dependent Material Properties

Research Article

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Abstract

This study is concerned with the aeroelastic behavior of a laminated composite rectangular plate with temperature dependent material properties. Plate equations for homogenous linear elastic material and small deformations are derived in the frame of the Kirchhoff theory. Uniform and linear temperature distributions are considered on the layered composite plate and it is assumed that the material properties of fiber and matrix vary with the temperature. The aerodynamic forces are obtained by the piston theory. Equations of motion are derived in the variational form by the use of the Hamilton principle. The Equations are solved using the finite element method. The laminated composite plates are discretized with the Semi loof thin shell elements with eight nodes and a total of thirty-eight degrees of freedom. The free vibration results are in a good agreement with the results of literature. The effects of aspect ratio, temperature distribution and lamination on the flutter boundary have been examined. The flutter occurs at a high dynamic pressure for the plate with high aspect ratio. The high temperatures on the plate results in a decrease of the flutter boundary. The number of laminate for a constant plate thickness affects the flutter boundary until a certain laminate number.

Keywords: Flutter; Composite Plate; Piston Theory; Temperature-Dependent Materials.

Introduction

Panel flutter is a self-excited oscillating phenomenon and involves interactions among elastic, inertia, and aerodynamic forces in supersonic flow. The panel flutter differs from wing flutter only in that the aerodynamic force resulting from the airflow acts only on one side of the panel. In the framework of small deflection linear structural theory, there is a critical speed of the airflow (or dynamic pressure λ_{cr}) beyond which the panel motion becomes unstable and grows exponentially with time.

Plate/shell panels are a popular and a useful form of structural components with significant applications in aerospace vehicles, such as high-speed aircrafts, rockets and spacecrafts. Future design concepts civil and military aircrafts are likely to imply the achievement of extremely light weight structural configurations. Therefore these panels are being constructed using advanced fiber-reinforced composite materials to achieve minimum weight design. When a flight vehicle travels at high supersonic speeds, it

only will experience flutter due to dynamic pressure but also will be affected by increased temperature owing to the aerodynamic heating. Structural components and/or mechanical elements, such as high-speed aircraft and spacecraft components, subjected to thermal loads due to high temperature, high gradient temperature, and cyclical changes of temperature, etc. The usage of new types of materials such as fiber-reinforced composite materials and functionally gradient materials is on the increase. It has become important to perform more accurate analyses of the thermo mechanical behavior of the above structures, elements and materials. The influence of temperature dependent material properties on thermal stresses at elevated temperature and/or high gradient temperature is quite significant.

The research progress and some of the references can be found in the literature as can be seen in Fung [1] and Dowell [2, 3]. Bismarck-Nasr [4] reviewed the finite element analysis of aeroelasticity of plates and shells. The most used aerodynamic theory is piston theory that was first suggested by Ashley and Zartarian [5]. Mei [6] developed a finite element approach to panel flut-

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ter. Dowell and Voss [7] studied on theoretical and experimental panel flutter. Rosettes and Tong [8] applied a hybrid stress finite element method and used linearized piston theory. Their results denoted that flutter characteristics are strongly dependent on the composite fiber angle and anisotropy. Srinivasan and Babu [9] studied the panel flutter of laminated composites by using the integral equations method. Wang [10] studied high supersonic/hypersonic flutter of prismatic composite plate/shell panels.

The effect of temperature on panel flutter behavior is presented by some researchers. Liaw [11] studied the geometrically nonlinear supersonic flutter characteristics of laminated composite thin-plate structures subjected to thermal loads. Lee et al. [12] investigated vibration and flutter analysis of stiffened composite plate considering thermal effect. Guo and Mei [13] presented a finite element time domain formulation using aeroelastic modes for the analysis of nonlinear flutter of isotropic and composite panels at arbitrary supersonic yawed angle and elevated temperature environment. Sohn and Kim [14] and Ibrahim et al. [15, 16] investigated thermal buckling and nonlinear flutter behaviors of functionally graded panels subjected to combined thermal and aerodynamic loads. Sohn and Kim [14] also analyzed the effects of volume fraction distributions, boundary conditions, temperature changes and aerodynamic pressures on panel flutter characteristics. Xue and Mei [17] presented a finite element formulation for analysis of large amplitude limit-cycle oscillations of panels with the influence of non-uniform temperature distribution. Librescu et al. [18] analyzed flutter and post-flutter behavior of long flat panels in a supersonic/hypersonic flow field exposed to a high-temperature field. Abbas et al. [19] investigated nonlinear flutter of isotropic and specially orthotropic panels in supersonic airflow under aerodynamic heating.

Only a few investigations have dealt with the temperature dependent material properties on panel/shell behavior. Mecitoğlu [20] denoted the results of the free vibrations of a conical shell with temperature dependent material properties. Prakash and Ganapathi [21] investigated numerically the influence of thermal environment on the supersonic flutter behavior of flat panels made of functionally graded materials using the finite element procedure. Olson [22] presented an analysis of the supersonic flutter of a finite circular cylindrical shell with temperature dependent material properties.

Therefore, in this study, the aeroelastic behavior of a laminated composite rectangular plate with temperature dependent mate-

rial properties was considered. The equations are solved using the finite element method. The laminated composite plates are discretized with the Semiloof thin shell elements with eight-nodes and a total of thirty-two degrees of freedom. The effects of aspect ratio, temperature distribution and lamination on the flutter boundary have been examined.

Governing Equations

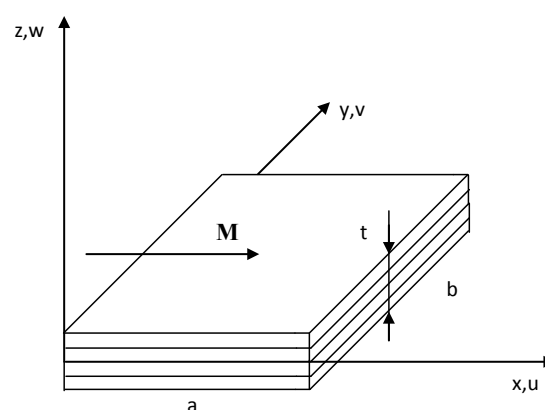
An exhaustive formulation of the analytical method within the linear elastic and aerodynamic theories can be found in Dowell's monograph [3]. The following considerations show the main steps in modeling the aerothermoelastic problem and its formulation within the finite element method.

Figure 1 defines the coordinate system to be used in developing the laminated aeroelastic plate analysis. The Cartesian coordinate system is assumed to have its origin on the middle surface of the plate, so that the middle surface lies in the xy plane. The displacements at a point in the x , y , z directions are u , v , and w , respectively. a and b are the dimensions of the rectangular plate, t is the plate thickness and M is the Mach number. The basic assumptions relevant to the laminated composite plate are [23]:

1. The plates consist of orthotropic laminae bonded together, with the principal material axes of the orthotropic laminae oriented along arbitrary directions with respect to the xy axes.
2. The thickness of the plate, t , is much smaller than the lengths along the plate edges, a and b .
3. The displacements, u , v , and w are small compared with the plate thickness.
4. The in-plane strains ϵ_x , ϵ_y and γ_{xy} are small compared with unity.
5. Transverse shear strains γ_{xz} and γ_{yz} are negligible.
6. Tangential displacements u and v are linear functions of the z coordinate.
7. The transverse normal strain ϵ_z is negligible.
8. Each ply obeys Hooke's law.
9. The plate thickness t is constant.
10. Transverse shear stresses τ_{xz} and τ_{yz} vanish on the plate surfaces defined by $z = \pm t/2$.

Assumption 5 is a result of the assumed state of plane stress in each ply, whereas assumptions 5 and 6 together define the Kirchhoff deformation hypothesis that normals to the middle surface remain straight and normal during deformation. According to the

Figure 1. Geometry of the aeroelastic plate.



Weierstrass theory (also shown in reference [24]), and assumptions 6 and 7, the displacements can be expressed as:

$$\begin{aligned} u(x, y, z, t) &= u^0(x, y, t) + z\beta_x(x, y, t) \\ v(x, y, z, t) &= v^0(x, y, t) + z\beta_y(x, y, t) \\ w(x, y, z, t) &= w^0(x, y, t) + w(x, y, t) \end{aligned} \quad (1)$$

Where u^0 and v^0 are the tangential displacements of the middle surface along the x and y directions, respectively. Due to assumption 7, the transverse displacement at the middle surface, $w^0(x, y)$, is the same as the transverse displacement of any point having the same x and y coordinates, so $w^0(x, y) = w(x, y)$. Substituting Equation (1) in the strain-displacement equations for the transverse shear strain and using assumption 5, we find that:

$$\beta_x(x, y) = -\frac{\partial w}{\partial x} \quad \beta_y(x, y) = -\frac{\partial w}{\partial y} \quad (2)$$

Substituting Equations (1) and (2) in the strain-displacement relations for the in-plane strains, we find that:

$$\begin{aligned} \epsilon_x &= \frac{\partial u}{\partial x} = \epsilon_x^0 + z\kappa_x \\ \epsilon_y &= \frac{\partial v}{\partial y} = \epsilon_y^0 + z\kappa_y \\ \gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \gamma_{xy}^0 + z\kappa_{xy} \end{aligned} \quad (3)$$

Where the strains on the middle surface are:

$$\epsilon_x = \frac{\partial u^0}{\partial x} \quad \epsilon_y = \frac{\partial v^0}{\partial y} \quad \gamma_{xy} = \frac{\partial u^0}{\partial y} + \frac{\partial v^0}{\partial x} \quad (4)$$

And the curvatures of the middle surface are:

$$\kappa_x = -\frac{\partial^2 w}{\partial x^2} \quad \kappa_y = -\frac{\partial^2 w}{\partial y^2} \quad \kappa_{xy} = -2\frac{\partial^2 w}{\partial x \partial y} \quad (5)$$

κ_x is a bending curvature associated with bending of the middle surface in the xz plane and κ_y is a bending curvature associated with bending of the middle surface in the yz plane. κ_{xy} is a twisting curvature associated with out-of-plane twisting of the middle surface, which lies in the xy plane before deformation.

Since Equation (3) gives the strains at any distance z from the middle surface, the stresses along arbitrary xy axes in the kth lamina of a laminate may be found by substituting Equation (3) into the lamina stress-strain relationships considering thermal effects as follows:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_k \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix}_k - \begin{Bmatrix} \epsilon_{x0} \\ \epsilon_{y0} \\ \gamma_{xy0} \end{Bmatrix}_k \quad (6)$$

Where the subscript k refers to the kth lamina. \bar{Q}_{ij} are the components of the transformed lamina stiffness matrix which are defined as follows:

$$\begin{aligned} \bar{Q}_{11} &= Q_{11} \cos^4 \theta + Q_{22} \sin^4 \theta + 2(Q_{12} + 2Q_{66}) \sin^2 \theta \cos^2 \theta \\ \bar{Q}_{12} &= (Q_{11} + Q_{22} - 4Q_{66}) \sin^2 \theta \cos^2 \theta + Q_{12} (\cos^4 \theta + \sin^4 \theta) \\ \bar{Q}_{22} &= Q_{11} \sin^4 \theta + Q_{22} \cos^4 \theta + 2(Q_{12} + 2Q_{66}) \sin^2 \theta \cos^2 \theta \\ \bar{Q}_{16} &= (Q_{11} - Q_{12} - 2Q_{66}) \cos^3 \theta \sin \theta - (Q_{22} - Q_{12} - 2Q_{66}) \cos \theta \sin^3 \theta \\ \bar{Q}_{26} &= (Q_{11} - Q_{12} - 2Q_{66}) \cos \theta \sin^3 \theta - (Q_{22} - Q_{12} - 2Q_{66}) \cos^3 \theta \sin \theta \\ \bar{Q}_{66} &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66}) \sin^2 \theta \cos^2 \theta + Q_{66} (\sin^4 \theta + \cos^4 \theta) \end{aligned} \quad (7)$$

Where θ is the lamina orientation angle and the Q_{ij} are the components of the lamina stiffness matrix, which are related to the engineering constants by:

$$\begin{aligned} Q_{11} &= \frac{E_1}{1 - \nu_{12}\nu_{21}} \\ Q_{12} = Q_{21} &= \frac{\nu_{12}E_2}{(1 - \nu_{12}\nu_{21})} \\ Q_{22} &= \frac{E_2}{1 - \nu_{12}\nu_{21}} \\ Q_{66} = G_{12} &= \frac{E_1}{2(1 + \nu_{21})} \end{aligned} \quad (8)$$

Where E_1 and E_2 longitudinal modulus of elasticity associated with the x and y direction, respectively. G_{12} is the shear modulus associated with the xy plane and ν_{12} is the Poisson's ratio.

We can write the constitutive relations as the matrix expressions:

$$\{\sigma\}_k = [\bar{Q}]_k \left\{ \{\epsilon\}_k - \{\epsilon_0\}_k \right\}$$

And in the case of thermal strains

$$\{\epsilon_0\} = \begin{Bmatrix} \epsilon_{x0} \\ \epsilon_{y0} \\ \gamma_{xy0} \end{Bmatrix} = \alpha \Delta T \begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix}$$

The force and moment resultants, per unit length, are defined as:

$$\begin{aligned} N_{\Delta T} &= \int_{-t/2}^{t/2} \{\sigma\}_k \Delta T dz \\ M_{\Delta T} &= \int_{-t/2}^{t/2} \{\sigma\}_k z \Delta T dz \end{aligned} \quad (9)$$

Which lead to the constitutive relations for a laminated panel: Here, refers to the kth lamina's stress.

$$\begin{Bmatrix} \{N\} \\ \{M\} \end{Bmatrix} = \begin{bmatrix} [A] & [B] \\ [B] & [D] \end{bmatrix} \begin{Bmatrix} \{\epsilon\} \\ \{\kappa\} \end{Bmatrix} - \begin{Bmatrix} N_{\Delta T} \\ M_{\Delta T} \end{Bmatrix} \quad (10)$$

{N} and {M} are the force and moment resultants with temperature dependent material properties, respectively. The laminate stiffness matrices are given by:

$$\begin{aligned}
 A_{ij} &= \int_{-t/2}^{t/2} (\bar{Q}_{ij})_k dz = \sum_{k=1}^N (\bar{Q}_{ij})_k (z_k - z_{k-1}) \\
 B_{ij} &= \int_{-t/2}^{t/2} (\bar{Q}_{ij})_k z dz = \frac{1}{2} \sum_{k=1}^N (\bar{Q}_{ij})_k (z_k^2 - z_{k-1}^2) \\
 D_{ij} &= \int_{-t/2}^{t/2} (\bar{Q}_{ij})_k z^2 dz = \frac{1}{3} \sum_{k=1}^N (\bar{Q}_{ij})_k (z_k^3 - z_{k-1}^3)
 \end{aligned}
 \tag{11}$$

The effects of elevated temperatures on the aeroelastic behavior of high-speed aircraft structures and launch vehicles as an especially important design consideration that needs further investigation and development. In the present study, considering elasticity modulus respect to temperature with linear approach, can be written as:

$$\begin{aligned}
 E_1(T) &= \frac{1}{T_2 - T_1} [(E_{1T_1} T_2 - E_{1T_2} T_1) + (E_{1T_2} - E_{1T_1}) T] \\
 E_2(T) &= \frac{1}{T_2 - T_1} [(E_{2T_1} T_2 - E_{2T_2} T_1) + (E_{2T_2} - E_{2T_1}) T] \\
 G_{12}(T) &= \frac{1}{T_2 - T_1} [(G_{12T_1} T_2 - G_{12T_2} T_1) + (G_{12T_2} - G_{12T_1}) T]
 \end{aligned}
 \tag{12}$$

Where T_1 is the room temperature and T_2 is 180°C. E_{1T_1} and E_{1T_2} are longitudinal modulus of elasticity associated with the direction 1 at temperature T_1 and T_2 , respectively. E_2 , which is the longitudinal modulus of elasticity associated with the direction 2 and shear modulus G_{12} can be written as the same. Temperature T is respect to x and can be written as:

$$T(x) = T_L + \frac{T_T - T_L}{a} x
 \tag{13}$$

Where T_L and T_T temperatures at leading ($x=0$) and trailing ($x=a$) edges.

The derivation of the aeroelastic equations is obtained using Hamilton's principle. A system of equations of motion for a nonconservative elastic system can be obtained using a variation of the form:

$$\int_{\tau_0}^{\tau_1} \delta(T - U) d\tau + \int_{\tau_0}^{\tau_1} \delta W d\tau = 0
 \tag{14}$$

where T is the kinetic energy, U is the potential energy of the system, and δW is the virtual work done by the aerodynamic forces acting on the structure from time τ_0 to τ_1 . Potential and kinetic energy can be expressed as:

$$U = \frac{1}{2} \int_V \{\sigma\}^T \{\varepsilon\} dV
 \tag{15}$$

$$T = \frac{1}{2} \rho \int_V (\dot{u}^2 + \dot{v}^2 + \dot{w}^2) dV
 \tag{16}$$

In the analysis of Reference [25], only rotary inertias of the faces about the elastic axis were considered, force reads,

$$W = - \int_A \frac{2\bar{q}}{V\beta} \left(V \frac{\partial w}{\partial x} + \frac{M^2 - 2}{M^2 - 1} \frac{\partial w}{\partial \tau} \right) w dA
 \tag{17}$$

Where $\bar{q} = \rho V^2 / 2$ is the free stream dynamic pressure,

$\beta = \sqrt{M^2 - 1}$, V is the free stream velocity, M is the free stream Mach number, and is ρ the air density. For sufficiently high Mach number, the expression (17) can be approximated as:

$$W = - \int_A \left(\frac{2\bar{q}}{M} \frac{\partial w}{\partial x} + \frac{2\bar{q}}{VM} \frac{\partial w}{\partial \tau} \right) w dA
 \tag{18}$$

The expression of the aerodynamic load given in (17) is known in the literature as the quasi-steady case. Further, if the aerodynamic damping is neglected in Equation (18), we get the quasi-static Ackeret's expression,

$$W = - \int_A \frac{2\bar{q}}{M} \frac{\partial w}{\partial x} w dA
 \tag{19}$$

The boundary conditions for a rectangular plate as shown in Figure 1 can be written as;

For clamped edges:

$$\text{At } x = \bar{x}; w = 0 \text{ and } \frac{\partial w}{\partial x} = 0
 \tag{20}$$

$$\text{At } y = \bar{y}; w = 0 \text{ and } \frac{\partial w}{\partial y} = 0
 \tag{21}$$

For simply supported edges:

$$\text{At } x = \bar{x}; w = 0 \text{ and } \frac{\partial^2 w}{\partial x^2} = 0
 \tag{22}$$

$$\text{At } y = \bar{y}; w = 0 \text{ and } \frac{\partial^2 w}{\partial y^2} = 0
 \tag{23}$$

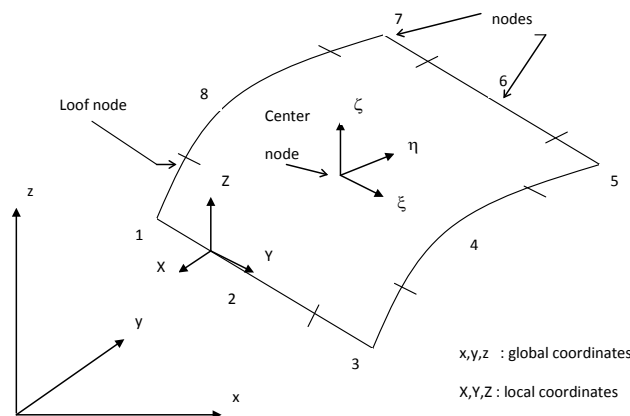
Method of Solution

In the present study, Semiloof shell element is chosen as a finite element for the solution. As shown in Figure 2, Semiloof element has 9 nodes and a total of 45 degrees of freedom. By applying shear constraints [26], it becomes a finite element model which has 8 nodes and a total of 32 degrees of freedom.

Displacement vectors at global and local coordinates can be defined as:

$$\{q_G\} = \begin{Bmatrix} u(x, y, z, \tau) \\ v(x, y, z, \tau) \\ w(x, y, z, \tau) \end{Bmatrix} \quad \{q_L\} = \begin{Bmatrix} U(X, Y, Z, \tau) \\ V(X, Y, Z, \tau) \\ W(X, Y, Z, \tau) \end{Bmatrix}
 \tag{24}$$

Figure 2. Semiloof element.



In general case, displacements, strains and stresses can be written in matrix form and substituting into Hamilton's principle (14), and minimizing the functional, the following matrix equation can be obtained for each element:

$$[k^e]\{Q^e\} + [m^e]\{\ddot{Q}^e\} + \bar{\lambda}[a^e]\{Q^e\} = \{0\} \quad \text{--- (25)}$$

Where $\bar{\lambda}$ is dynamic pressure parameter and defined as:

$$\bar{\lambda} = 2\bar{q} / M \quad \text{--- (26)}$$

Where \bar{q} is the free stream dynamic pressure. $[k^e]$, $[m^e]$ and $[a^e]$ are the element stiffness, mass and aerodynamic stiffness matrices, respectively. For whole system, applying boundary conditions and using standard assembly technique for the finite element method,

$$[K]\{Q\} + [M]\{\ddot{Q}\} + \bar{\lambda}[A]\{Q\} = \{0\} \quad \text{--- (27)}$$

Can be obtained. $[K]$, $[M]$ and $[A]$ are the system stiffness, mass and aerodynamic stiffness matrices, respectively. $\{Q\}$ is the vector of the system nodal degrees of freedom. The system of Equation (27) assumes solutions in the form,

$$\{Q\} = e^{-i\omega t} \{Q_0\} \quad \text{--- (28)}$$

Considering ω complex number, the Equation (28) reads

$$\{Q\} = e^{\omega t} [\cos(\omega_R t) + i \sin(\omega_R t)] \{Q_0\} \quad \text{--- (29)}$$

Substituting Equation (29) into the Equation (27)

$$[\omega^2 [M] + [K] + [A]]\{Q\} = \{0\} \quad \text{--- (30)}$$

the problem becomes an eigenvalue problem. Therefore, the determinant of the expression (30) has to be zero also shown in Equation (31).

$$|[K] + [A] + \omega^2 [M]| = 0 \quad \text{--- (31)}$$

ω^2 Correspond to the squares of natural frequencies of the plate. The flutter occurs when any two natural frequencies coalesce where is purely an imaginary number.

Numerical Results

A general computer program was developed for the present composite shell finite element formulation as applied to supersonic panel flutter analysis. As part of the evaluation process, the natural frequencies were first obtained and compared with Srinivasan and Babu's [9] study. Unsymmetrically laminated composite two-layered square plate with all edges clamped (also shown in Figure 3) has been studied by Srinivasan and Babu [9]. The material properties are $E_1 = 213.8 \text{ GPa}$, $E_2 = 18.6 \text{ GPa}$, $G_{12} = 5.165 \text{ GPa}$, $\nu_{12} = 0.28$ and $\rho = 1920 \text{ kg/m}^3$.

The dimensionless dynamic pressure parameter and flutter frequency, respectively, defined as:

$$\frac{a}{D} \frac{n\bar{q}}{M} \kappa = \omega^2 \frac{\rho t a^4}{D} \quad \text{--- (32)}$$

where $D = E_1 t^3 / [12(1 - \nu_{12}\nu_{21})]$ also known as flexure rigidity. n equals to one when the airflow acts only on one side of the panel. If the airflow acts on two sides, n equals to two.

For convergence study, the natural frequencies obtained in the present investigation have been compared with those given by Srinivasan and Babu [9]. The authors' results agree well with this study, as can be seen from Table 1.

The material properties for the main model which the authors will investigate in this paper are $E_1 = 62 \text{ GPa}$, $E_2 = 24.8 \text{ GPa}$, $G_{1,2} = 17.38 \text{ GPa}$, $\nu_{1,2} = 0.23$ and $\rho = 2000 \text{ kg/m}^3$ Firstly, this square plate considered $t/a = 0.01$, two-layered and meshed 5×5 , 7×7 and 10×10 . The natural frequencies obtained and compared with ANSYS results. Present results agree well with ANSYS, as can be seen from Table 2. Using a 10×10 mesh, a well agreement is found. Thus, a 10×10 mesh was used to model the square plates. The

Figure 3. Aeroelastic two-layered composite plate.

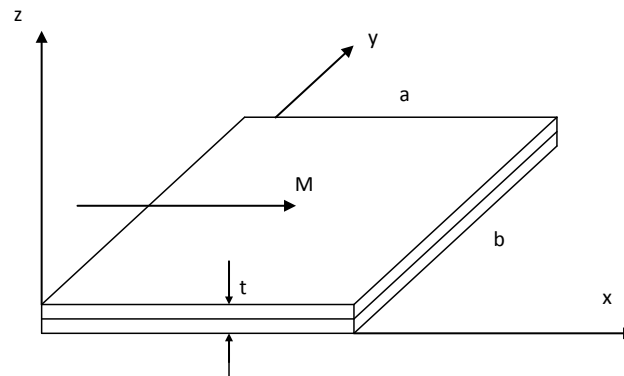


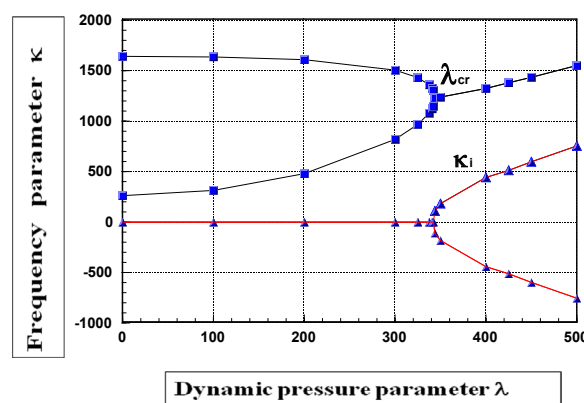
Table 1. Natural frequencies of all edges clamped rectangular plate (Hz).

Frequency	Present Study	Srinivasan and Babu	Srinivasan and Babu
		IE Method	Series method
1	31.05	33.82	33.71
2	51.88	46.35	46.14
3	63.01	63.84	62.22
4	63.17	65.39	62.36
5	78.59	73.96	71.19

Table 2. Natural frequencies of all edges clamped rectangular plate compared with ANSYS (Hz).

Frequency	Present Study (Semilo of Element)			ANSYS
	5x5	7x7	10x10	10x10
1	76.088	76.106	76.131	77.340
2	148.274	149.176	150.027	157.33
3	153.716	155.225	156.483	157.33
4	228.161	227.436	227.467	234.17
5	257.721	259.907	263.132	280.48
6	272.151	274.006	278.121	282.06

Figure 4. Flutter analysis of eight-layered composite square plate with all edges simply supported.



flutter boundaries determined for an [0/90/0/90/0/90/0/90] eight-layered, composite square plate with all edges simply supported. Figure 4 shows the flutter boundary of an eight-layered composite square plate.

In the present study, the effects of lamination on the flutter boundary have been examined and critical resultants have been

determined for 4, 6, 8 and 12-layered composite square plates with all edges simply supported. The numbers of layer for a constant plate thickness affect the flutter boundary until a certain laminate number as shown in Table 3.

Table 4 shows the effect of aspect ratio on the flutter boundary for the angle-ply laminated composite square plates where $t/$

Table 3. The effect of layer numbers on the flutter boundary.

	4 Layers	6 Layers	8 Layers	12 Layers
λ_{cr}	368.75	343.75	343.75	344.53
κ_{cr}	1287.05	1232.39	1234.02	1235.92

Table 4. The effect of aspect ratio on the flutter boundary.

Aspect Ratio (a/b)	0.5	1.0	2.0
λ_{cr}	264.06	343.75	711.71
κ_{cr}	247.05	1234.02	3792.65

$a=0.01$. The flutter boundary increases with the aspect ratio as shown clearly in Figure 5. The first two mode shapes of the square plate that flutter occurs can be seen in Figure 6.

The influence of temperature dependent material properties on thermal stresses at elevated temperature and/or high gradient temperature is quite significant. Therefore, in this study, the aeroelastic behaviour of a laminated composite rectangular plate with temperature dependent material properties was considered. Elasticity modulus of fibers and resin decrease when the temperature increases. In the present study, natural frequencies and flutter analysis were obtained also considering this heating effect at high speeds. Table 5 and Table 6 show, respectively, natural vibration frequencies at the uniform and linearly varying elevated temperature. The analysis results denoted that high temperatures on the plate results in a decrease of the flutter boundary, as can be seen in Table 7. The flutter analyses have been done applying to leading and trailing edges different temperatures and considering the linearly varying temperature in the flow direction. The uniform and varying temperatures have similar effects on the flutter boundary as shown in Table 7 and 8.

Conclusion

The aeroelastic behaviour of a laminated composite rectangular plate with temperature dependent material properties was investi-

gated. Plate equations for homogenous linear elastic material and small deformations are derived in the frame of the Kirchhoff theory. Uniform and linear temperature distributions are considered on the layered composite plate and it is assumed that the material properties of fiber and matrix vary with the temperature. The aerodynamic forces are obtained by the piston theory. Equations of motion are derived in the variational form by the use of the Hamilton principle. The Equations are solved using the finite element method. The laminated composite plates are discretized with the Semiloof thin shell elements with eight nodes and a total of thirty-two degrees of freedom. A general computer program was developed for the present composite shell finite element formulation as applied to supersonic panel flutter analysis. The free vibration results compared with the results of literature and ANSYS software program. Present results agree well with ANSYS and literature also shown. The effects of aspect ratio, temperature distribution and lamination on the flutter boundary have been examined. The flutter occurs at a high dynamic pressure for the plate with high aspect ratio. The high temperatures on the plate results in a decrease of the flutter boundary. The number of laminate for a constant plate thickness effects the flutter boundary until a certain laminate number.

The present results illustrate the effects of aspect ratio, temperature and number of layers on the flutter boundary. Extensions can also be made to include aerodynamic and structural nonlin-

Figure 5. The effect of aspect ratio on the flutter boundary.

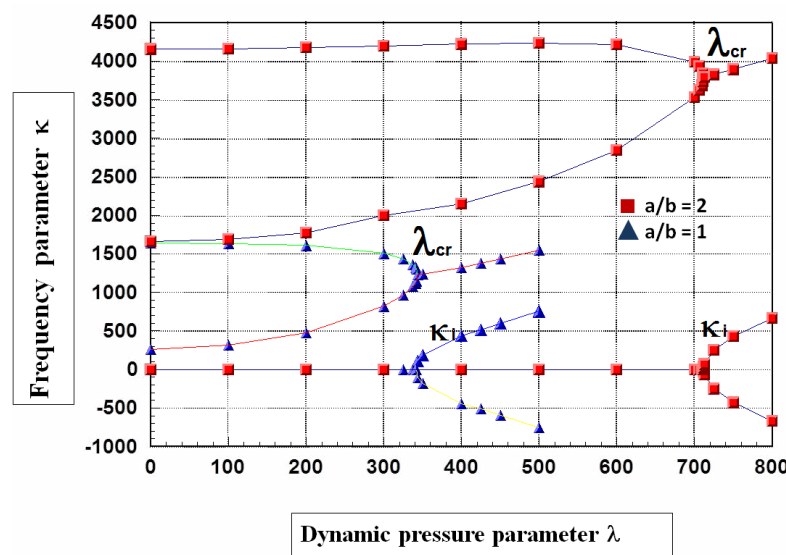


Figure 6. Mode shapes for the first two modes.

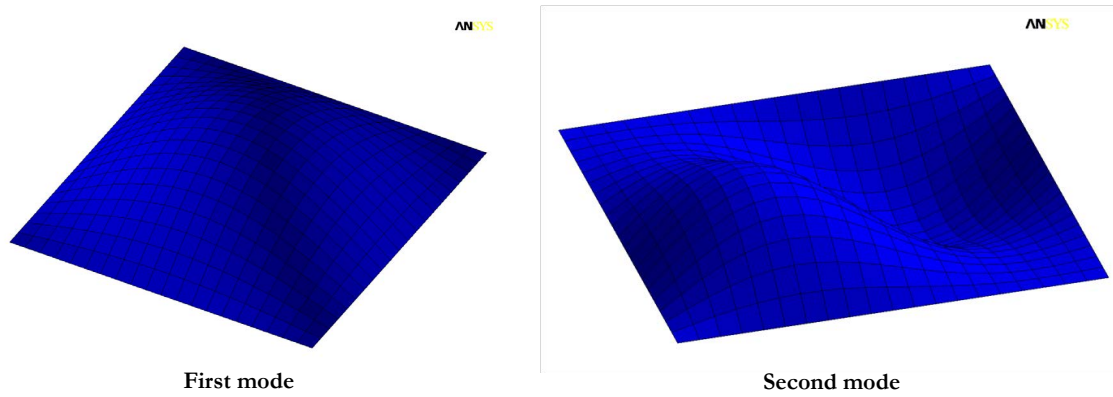


Table 5. Natural free vibration frequencies at the uniform temperatures.

Frequency	1	2	3	4
	T = 20°C	T = 100°C	T = 180°C	T = 300°C
1	261.089	239.249	217.409	184.647
2	1641.667	1511.595	1381.466	1186.115
3	1646.729	1516.705	1386.641	1191.431
4	4177.175	3827.375	3477.543	2952.711
5	6526.139	6028.622	5530.555	4781.943
6	6559.975	6062.69	5564.951	4817.054

Table 6. Natural free vibration frequencies at linearly varying elevated temperatures.

Frequency	T _L = 200°C	T _L = 300°C	T _L = 400°C
	T _T = 100°C	T _T = 150°C	T _T = 200°C
1	204.634	236.135	184.051
2	1302.134	1492.871	1182.298
3	1306.963	1497.051	1185.969
4	3272.929	3777.107	2942.539
5	5217.969	5956.214	4765.942
6	5247.757	5981.742	4786.305

Table 7. Flutter boundary at the uniform temperatures.

	T = 20°C	T = 100°C	T = 180°C	T = 300°C
λ_{cr}	343.75	317.18	289.84	248.49
K_{cr}	1234.02	1134.28	1031.26	881.04

Table 8. Flutter boundary at linearly varying elevated temperatures.

	T _L = 200°C	T _L = 300°C	T _L = 400°C
	T _T = 100°C	T _T = 150°C	T _T = 200°C
λ_c	270.31	312.51	247.65
K_{cr}	969.63	1117.23	878.62

erities. The effects of aerodynamic damping, nonlinear temperature gradients and moisture on the flutter boundary can also be investigated in further studies.

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